Experimental scheme for quantum teleportation of a one-photon packet

S. N. Molotkov
Institute of Solid-State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Moscow Region, Russia

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A complete protocol and an optical scheme for experimental implementation of the quantum teleportation of an unknown one-photon wave packet are proposed. © *1998 American Institute of Physics.* [S0021-3640(98)01715-0]

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Quantum mechanics forbids the cloning (copying) of an unknown quantum state (the no cloning theorem[1]). Is it possible to transmit to a distant user an a priori unknown quantum state without transmitting the state itself? Any measurement performed to obtain classical information about the state for subsequent transmission of this information, generally speaking, changes the state itself without giving complete information about it. The preparation of multiple copies followed by measurements on the copies to obtain exhaustive information is forbidden by the no cloning theorem. Therefore it is impossible to transmit information about an unknown quantum state using only transmission of information along a classical communication channel.

Quantum teleportation removes this restriction, if a quantum communication channel is used in addition to a classical channel. The idea of quantum teleportation for the case of quantum states described by a discrete variable, specifically, for a spin-1/2 particle in an unknown state, was advanced in Ref. 3. Nonlocal EPR[a)] correlations are used as the quantum channel.[2,3] The EPR pair is a pair of particles in an entangled state. Entanglement entails a special type of quantum correlations which have no classical analog.

The quantum teleportation protocol proposed in Ref. 4 is as follows. To teleport an *a priori* unknown quantum state which user A possesses to a distant user B, user A generates an EPR pair. One particle of the EPR pair remains with A, while the second particle is directed toward the distant user B. User A performs on the remaining second particle of the EPR pair and on the particle in the unknown state a simultaneous measurement whose outcome yields classical information. Since nonlocal quantum correlations exist initially in the EPR pair, the measurement attaches the state of the unknown particle to the second particle of the EPR pair at the user B. The state of the second particle is identical, to within a unitary rotation, to the unknown state. The classical information obtained in the measurement is transmitted by user A to user B and indicates to user B which unitary transformation must be performed in order to obtain the new

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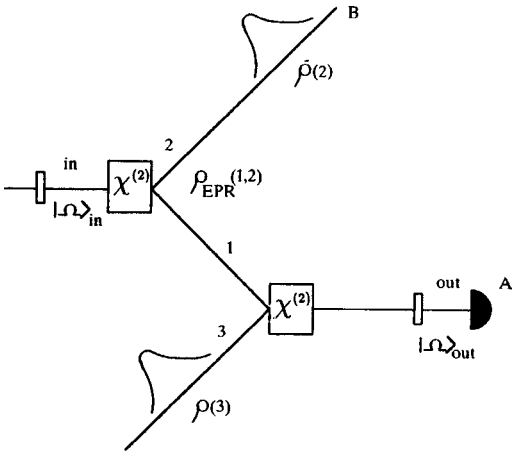

FIG. 1.

state, which will now be an exact copy of the initial unknown state. In the teleportation process the user A does not obtain any information about the unknown teleported state.

Quantum teleportation was recently demonstrated experimentally for a photon in a state with unknown polarization.[5,6]

The question of teleportation of the wave function of a particle in the one-dimensional case, where the continuous dynamical variables are the coordinate and momentum, was investigated in Ref. 7, where a wave function of the form in Ref. 2 was used as the wave function of the EPR pair. Quantum teleportation of a quantum state described by the dynamical variables (x,p) for the case of nonideal EPR correlations was investigated in a recent work[8] (the unknown state in Ref. 8 corresponds to a one-mode state of a photon). A quadratically squeezed state was used as the EPR state, while the homodyne detection procedure essentially corresponded to a measurement.

In the present letter a new scheme (the complete protocol and experimental implementation) is proposed for teleportation of a multimode state — a one-photon wave packet — using an EPR pair in an entangled state in terms of energy–time variables.

To further simplify the formulas we shall assume that the polarization state of the packet is known. The following arguments are also valid for the case of unknown polarization, which can be taken into account by introducing an additional index. The state of a one-photon wave packet can be represented in the form (see, for example, Ref. 9)

$$|1\rangle_3=\int_0^\infty d\omega\, f(\omega)\hat{a}^+(\omega)|0\rangle=\int_0^\infty d\omega\, f(\omega)|\omega\rangle_3, \tag{1}$$

$$[\hat{a}(\omega),\hat{a}^+(\omega')]=I\delta(\omega-\omega'), \quad \int_0^\infty |f(\omega)|^2 d\omega=1,$$

where $\hat{a}^+(\omega)$ and $\hat{a}(\omega)$ are creation and annihilation operators of a one-mode Fock state $|\omega\rangle_3$, $|0\rangle$ is the vacuum state, and $f(\omega)$ is the amplitude of the packet. The index 3 indicates the number of the channel (see Fig. 1). The density matrix at an arbitrary

moment in time has the form

$$\rho(3)=\left(\int_0^\infty d\omega e^{-i\omega t}f(\omega)|\omega\rangle_3\right)\left(\int_0^\infty d\omega_3'\langle\omega|e^{i\omega' t}f^*(\omega')\right). \tag{2}$$

The state of the EPR photon pair in our case can be represented in the form (similar EPR photon pairs are obtained in downward (in energy) parametric conversion processes[10])

$$|\psi_{\rm EPR}\rangle_{1,2}=\int_0^\infty d\omega|\omega\rangle_1\otimes|\Omega-\omega\rangle_2, \quad \rho_{\rm EPR}(1,2)=|\psi_{\rm EPR}\rangle_{1,2\ \ 1,2}\langle\psi_{\rm EPR}|, \tag{3}$$

where Ω is the pump frequency and the indices 1, 2 enumerate the channels (see Fig. 1). The normalization of the state (3) is immaterial for what follows.

According to the general scheme of Refs. 11–13, quantum-mechanical measurements are described by positive operators which realize a decomposition of unity. The measurements of observables, which correspond to self-conjugate operators, correspond to orthogonal decompositions of unity. The parameters (time, rotation angle) do not correspond to any self-conjugate operators, so that such measurements are described by nonorthogonal decompositions of unity.[11–13]

The idea of teleportation in application to the present case consists in using entangled measurements of energy-time for a pair of photons, one from an EPR pair and a photon in an unknown state. A measurement is given by a nonorthogonal decomposition of unity:[14]

$$\int\int M(dtd\Omega_+)=\int\int R^+R\,(dtd\Omega_+)=I, \tag{4}$$

where R is the ''reduction'' operator, $M(dtd\Omega_+)$ describes a quantum operation and is a positive-definite operator measure — POVM (for the details see, for example, Refs. 15 and 16). We have[14]

$$M(dtd\Omega_+)=\left(\int d\omega_- e^{i\omega_- t}|\omega_+ +\omega_-\rangle_1\otimes|\omega_+ -\omega_-\rangle_3\right)$$
$$\times\left(\int d\omega_-' e^{-i\omega_-' t}{}_3\langle\omega_+ -\omega_-'|\otimes{}_1\langle\omega_+ +\omega_-'|\right)\frac{dtd\omega_+}{2\pi}, \tag{5}$$

where $\omega_\pm=\Omega_\pm/2$. The integration extends over the frequencies at which the arguments in the Fock states are positive. Note that the frequency ω_+ is a common frequency in the bra and ket states.

According to the general ideology of quantum-mechanical measurements,[10–13,15,16] the application of a quantum operation (measurement) to a system with density matrix ρ transfers the system into a new state

$$\rho\to R_i\rho R_i^+/{\rm Tr}\{R_i\rho R_i^+\}.$$

The probability of the i-th outcome is given by the formula ${\rm Pr}={\rm Tr}\{R_i\rho R_i^+\}$, where $E_i=R_i^+R_i$ is the POVM element.

In our case, after a measurement by user A the state of the second photon from the EPR pair at user B is given by the density matrix

$$\tilde{\rho}(2)=\frac{\mathrm{Tr}_{1,3}\{\rho_{\mathrm{EPR}}(1,2)\otimes\rho(3)M(dtd\Omega_+)\}}{\Pr\{dtd\Omega_+\}}, \tag{6}$$

$$\Pr\{dtd\Omega_+\}=\mathrm{Tr}_{1,2,3}\{\rho_{\mathrm{EPR}}(1,2)\otimes\rho(3)M(dtd\Omega_+)\}, \tag{7}$$

$$\tilde{\rho}(2)=\left(\int d\omega e^{-i(\Omega_+/2+\omega)t}f(\Omega_+-\omega)|\Omega-\omega\rangle_2\right)$$
$$\times\left(\int d\omega' e^{i(\Omega_+/2+\omega')t}{}_2\langle\Omega-\omega'|f^*(\Omega_+-\omega')\right). \tag{8}$$

Formally, the measurement (4) corresponds to the situation where the moment t of the measurement and the frequency Ω_+ are chosen by the experimenter, and the probability of a positive outcome of such a measurement is given by Eq. (7). Teleportation will be ideal if $\Omega_+=\Omega$ is chosen (the detection frequency ω_+ of the detector equals the pump frequency Ω). In this case, as follows from Eq. (6), the state $\tilde{\rho}(2)$ is identical to $\rho(3)$ to within a phase factor, which can be eliminated by the user B if A reports to B along a classical channel the moment t of detection. We note that if $\Omega_+=\Omega$, the moment t of detection does not depend on the unknown state $\rho(3)$.

Physically, the measurement (4) can be understood as follows. The user A possesses a continuum of detectors which are ''tuned'' to frequencies in the interval $(0, \infty)$ and each of which can be triggered at an arbitrary moment t in time, formally, from $(-\infty, +\infty)$. The probability that a detector at frequency Ω_+ is triggered at a certain moment t is given by formula (7). The triggering probability is time-independent (equally likely at any moment in time) only for one detector, tuned to the frequency Ω. Teleportation will be ideal only in the case that the detector at the frequency Ω is triggered. As follows from Eq. (7), the triggering probability does not depend on the unknown input state. The user A does not obtain any information about the teleported state.

Since measurements can be performed at spatially separated points, the times in the formulas should be understood to be the times presented minus the time of flight ($t\to t-x/c$). As will be seen below, this will not be important in the case of ideal teleportation.

The main difficulty in realizing teleportation of a one-phonon packet lies in realizing the measurement (4). The measurement (4) on a photon pair is intermediate between a measurement of time and frequency. An experimental implementation of this measurement is proposed below. The idea is to convert the photon pair into one photon on which a measurement is performed with an narrow-band photodetector. The latter can be realized experimentally quite simply.

The experimental arrangement is shown in Fig. 1. The first nonlinear crystal with second-order susceptibility χ and a narrow-band filter at frequency Ω serve to generate the EPR pair in channels 1 and 2. An unknown one-photon packet enters the channel 3. The one-photon packet can be prepared by exciting a two-level system with a π pulse in the distant past. The second nonlinear crystal, a narrow-band filter at frequency Ω following it, and then an ordinary photodetector implement the measurement (4). A teleported state appears in channel 2.

Let us consider in succession the evolution of the input states through the optical scheme. The state after the first narrow-band filter in front of the first nonlinear crystal is described by a monochromatic state with density matrix

$$\rho_{in}(in)=|\Omega\rangle_{in\,in}\langle\Omega|, \tag{9}$$

which can be obtained by using the first filter to cut a narrow band out of an auxiliary one-photon packet fed into the entrance in (Fig. 1). The interaction of the photons in the nonlinear crystal is described by the Hamiltonian in the interaction representation (see details in Refs. 17 and 18):

$$H_1(t)=\chi\int d\mathbf{x}E_{in}^{(+)}(\mathbf{x},t)E_1^{(-)}(\mathbf{x},t)E_2^{(-)}(\mathbf{x},t)+\text{ h.c.}, \tag{10}$$

where all nonessential constants are included in the definition of χ, which, as is ordinarily done,[17,18] will be assumed to be frequency-independent. For what follows, it is convenient to represent the electric-field operators in the form[9]

$$E_i^{(-)}(\mathbf{x},t)=\frac{1}{\sqrt{2\pi}}\int_0^\infty d\omega e^{i(\omega t-\mathbf{kx})}\hat{a}^+(\omega)|0\rangle_i=\frac{1}{\sqrt{2\pi}}\int_0^\infty d\omega e^{i(\omega t-\mathbf{kx})}|\omega\rangle_i, \tag{11}$$

where i is the channel number. We proceed similarly for $E_i^{(+)}=(\mathbf{x},t)$ also. Taking account of Eq. (11) we have

$$H_1(t)=\frac{\chi}{(2\pi)^{3/2}}\int\int\int d\omega_1 d\omega_2 d\omega_{in}e^{it(\omega_1+\omega_2-\omega_{in})}|\omega_1\rangle_1 \otimes|\omega_2\rangle_{2\,in}\left\langle \omega_{in}\right|\int_{vol}d\mathbf{x}e^{-i\mathbf{x}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_{in})}+\text{h.c.} \tag{12}$$

The second integral extends over the volume of the crystal. This gives a δ-function of the momenta, and leads to the phase matching condition[18] $(\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_{in})$, which we shall assume to hold (in what follows, this means that $\mathbf{k}_2\|\mathbf{k}_3$; see Fig. 1). Next, χ is taken to be the renormalized value taking account of the additional factors from the second integral. The first-order susceptibility, which is always present, can be neglected for our purposes, since the terms in the Hamiltonian which are associated with it do not contribute in the *out* channel.

The state after the first crystal in channels 1 and 2 is described by the density matrix

$$\rho_{\text{EPR}}(1,2)=S(t)\rho_{in}(in)S^{-1}(t), \tag{13}$$

where $S(t)$ is the S matrix

$$S(t)=e^{i\int_{-\infty}^{t}H_1(t')dt'}=1+S^{(1)}+S^{(2)}+\ldots\ . \tag{14}$$

In first order in χ we have

$$S^{(1)}=i\chi\int\int d\omega_1 d\omega_{in}|\omega_1\rangle_1\otimes|\omega_{in}-\omega_1\rangle_{2\,in}\langle\omega_{in}|+\text{h.c.} \tag{15}$$

The upper integration limit in the exponential in S can be replaced by ∞. Physically, this is related essentially with the fact that the entrance state is monochromatic (roughly

speaking, infinitely extended in time), and the teleportation process is formally stationary. To within the normalization, which is unimportant, the state in channels 1 and 2 can be described by the density matrix

$$\rho_{\mathrm{EPR}}(1,2)=\chi^2\left(\int_0^\infty d\omega|\omega\rangle_1\otimes|\Omega-\omega\rangle_2\right)\left(\int_0^\infty d\omega'{}_1\langle\omega'|\otimes_2\langle\Omega-\omega'|\right). \tag{16}$$

A measurement by the photodetector, in front of which a narrow-band at frequency Ω is installed, is formally described by the projector $P(\Omega)=|\Omega\rangle_{out\,out}\langle\Omega|$.

The teleported state in channel 2 after detection by the photodetector is described by the density matrix (once again, to within the normalization)

$$\tilde{\rho}(2)=\mathrm{Tr}_{out}\{S(t)\rho_{in}(in)\otimes\rho(3)S^{-1}(t)P(\Omega)\}, \tag{17}$$

where $S(t)$ is now the complete S matrix of the entire optical scheme:

$$S(t)=\exp\left\{i\int_{-\infty}^{t}[H_1(t')+H_2(t')]dt'\right\}=1+S^{(1)}+S^{(2)}+\ldots, \tag{18}$$

where $H(t)_2$ is the Hamiltonian for the second nonlinear crystal, which to within a replacement of the indices is similar to $H(t)_1$ in Eq. (12). The S-matrix terms of the form

$$S^{(2)}\propto\chi^2\left(\int\int d\omega_1 d\omega_{in}|\omega_1\rangle_1\otimes|\omega_{in}-\omega_1\rangle_{2\,in}\langle\omega_{in}|\right)$$
$$\times\left(\int\int d\omega_1' d\omega_{out}|\omega_{out}\rangle_{out\,1}\langle\omega_3'|\otimes_{in}\langle\omega_{out}-\omega_1'|\right) \tag{19}$$

contribute to the teleportation process. Taking account of Eq. (19), the density matrix in channel 2 is identical, to within the normalization, to the initial density matrix of the unknown wave packet:

$$\tilde{\rho}(2)=\chi^4\left(\int_0^\infty d\omega f(\omega)|\omega\rangle_3\right)\left(\int_0^\infty d\omega_3'\langle\omega|f^*(\omega')\right). \tag{20}$$

It also follows from Eq. (17) that the detection probability in the *out* channel does not depend on the unknown state and is proportional to

$$\mathrm{Pr}=\mathrm{Tr}_{2,out}\{S(t)\rho_{in}(in)\otimes\rho(3)S^{-1}(t)P(\Omega)\}\propto\chi^4. \tag{21}$$

In this scheme the classical channel is necessary to report to a distant user the fact that a photodetector has been triggered; in this case teleportation is considered to be successful. The probability (efficiency) of the teleportation process is small to the degree that χ^4 is small. On account of the terms in the S matrix which are of next higher-order in χ in the S matrix, the fraction of spurious photodetector triggerings for which an incorrect state will be teleported is small additionally with respect to the parameter χ^2. We note that formally the teleportation process is stationary (it lasts for an infinitely long time), since it is necessary to prepare a monochromatic input state. In this case the probability of detection by a photodetector in the *out* channel is the same at any moment in time and does not depend on the input state. It is obvious that in this case user A possesses zero information about the teleported state.

Of course, quantum teleportation does not permit transmitting information faster than the speed of light. In this scheme the intuitive and qualitative explanation reduces to the following. Since the input state is monochromatic and, as is always surmised intuitively, is nonlocalized (infinitely extended), the latter fact signifies that the field is seemingly ''preprepared'' in all space, including ''preprepared'' for the spatially distant users A and B also. A measurement by user A transfers the entire system into a new state — it reduces the state vector ''immediately'' and ''everywhere'' for the entire system. Usually, this part is regarded as intuitively unnacceptable. However, this ''immediately'' and ''everywhere'' does not make it possible to transmit information faster than the speed of light. To transmit classical information from A to B by means of a teleported state, a classical channel from A to B is required in order to report that the detector has been triggered and teleportation has taken place. The classical communication channel presupposes that a classical object is transmitted from A to B; the velocity of the object cannot exceed the velocity of light. The question of whether the field is ''preprepared'' is everywhere closely related with the question of photon localizability (more accurately, nonlocalizability) (see, for example, Refs. 19–23). As far as is known, this question has not yet been discussed in detail in the context of quantum teleportation.

We note that the teleportation process can be reformulated in terms of a diagrammatic technique. The averaging is performed not over the ground state, but rather over a ''stationary'' state $|\Omega\rangle\otimes\int_0^\infty d\omega f(\omega)|\omega\rangle$ — the input monochromatic state and the state of a one-phonon packet. In the case of ideal teleportation, such a state is also an output state. In this sense the process is stationary, and the averaging is performed with respect to this stationary state, which need not be the ground state. In this case the diagrammatic technique is constructed similarly to the way this is done in the Keldysh method.[24]

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[a)]The name originates from the well-known Einstein–Podolsky–Rosen effect.[2]

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